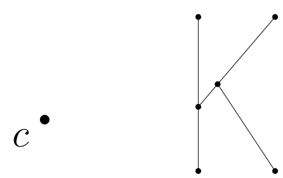
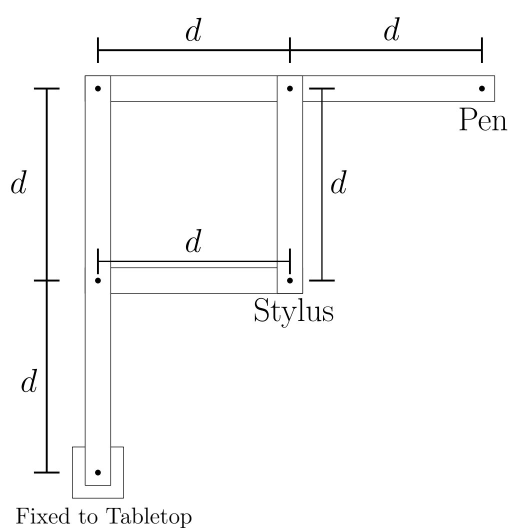
Question 1: In the video, we saw how to dilate a shape using just a pencil and straight-edge. Use this technique to dilate the letter K below by a factor of 2. Use point c as your center point for the dilation.



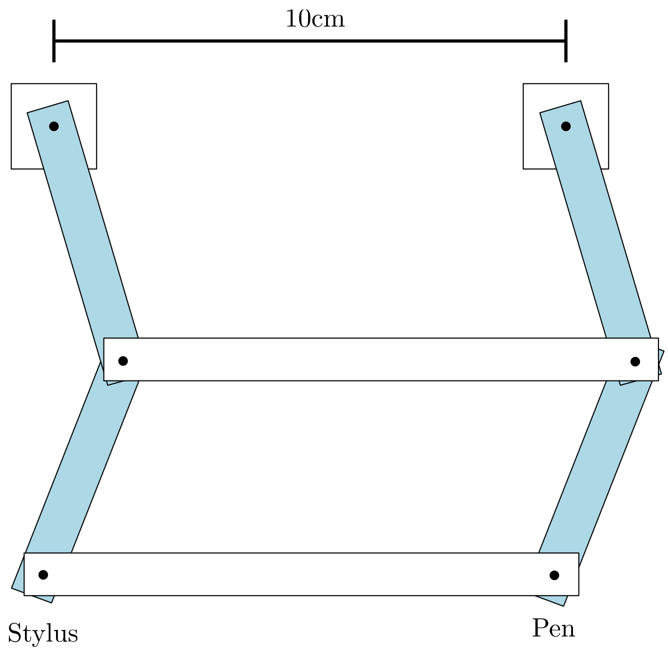
Question 2: In the video, we considered a pantograph that dilates shapes by a factor of 2. That machine had the following design:



Part 1: How would you change the pantograph to dilate shapes by a factor of 1/2? Justify your answer geometrically.

Part 2: How would you change the pantograph to dilate shapes by a factor of 3? Justify your answer geometrically.

Question 3: An inventor has presented you with the design for a drawing machine shown below. She claims that if you trace a shape with the stylus, the machine will draw a copy of the shape translated 10cm to the right. Explain why this machine performs a horizontal translation 10cm to the right.



You should assume:

- Dots represent hinges, and the two squares are fixed to the tabletop.
- On each of the horizontal white bars, the distance between the hinge points is 10 cm.
- The shaded bars are all of equal length.

Worksheet Key

Disclaimer: Depending on what your students already know, these answers might be too brief or too thorough. I mostly intend for them to give you an idea how I approached these problems.

Answer 1: This problem is straightforward to solve using the techniques seen in the video. The enlarged letter K should appear to the right of the original one, with all its lengths doubled.

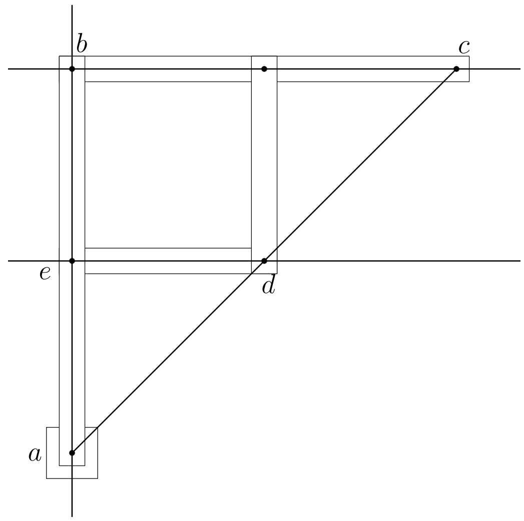
Students will need rulers to draw lines through point c and the points on the letter K. It might be a good idea to have colored pencils or pens available, so that the enlarged letter K can be distinguished from the lines through C.

Once students have drawn their six lines through c and the points on K, they will need to *measure the distance* from c to each point p, and calculate twice that distance to determine where to draw the new point. Measuring in centimeters can make the calculations simpler.

Answer 2: In the film, we saw the characters explain why their pantoraph performs dilation by a factor of 2. That explanation can be reused to solve both parts of this problem.

Judging whether an explanation is satisfactory depends on what assumptions one is allowed to make. Discussing these assumptions is an important part of real-world problem solving!

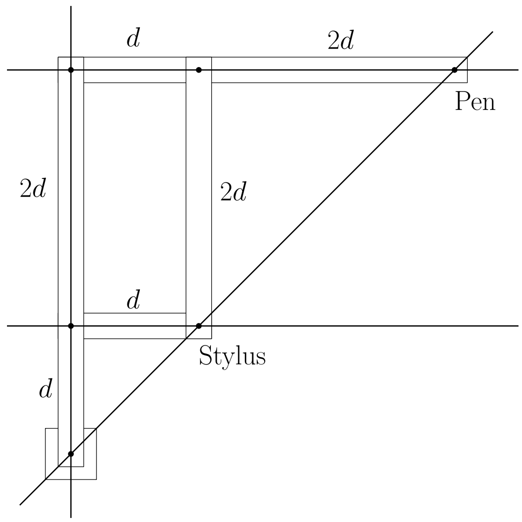
For part 1, I would start by drawing the lines below, so that we have two triangles: Δabc and Δaed. A reasonable starting assumption is that lines $\overline{bc}$ and $\overline{ed}$ are parallel, as was stated in the film.



The first thing students will need to do is to explain why Δaed and Δabc are similar triangles. Since $\overline{bc}$ and $\overline{ed}$ are parallel, the angles $\angle aed$ and $\angle abc$ are equal. Based on the side lengths shown in the diagram, the Side-Angle-Side criterion implies Δaed and Δabc similar. In fact, the side lengths imply that Δabc is obtained by scaling Δaed by a factor of two. So side length $\overline{ac}$ is twice as long as $\overline{ad}$. Next, we remember that the line through a and c is the line used to perform the dilation, with a as the center point.

When we wanted to dilate by a factor of 2 about a, we put the stylus at d and the pen at c, because $\overline{ac}$ is twice the length of $\overline{ad}$. So if we want to dilate by a factor of $1/2$ about a, we just need to put the stylus at c and the pen at d, because $\overline{ad}$ is half the length of $\overline{ac}$. In other words, we just exchange the locations of the stylus and pen.

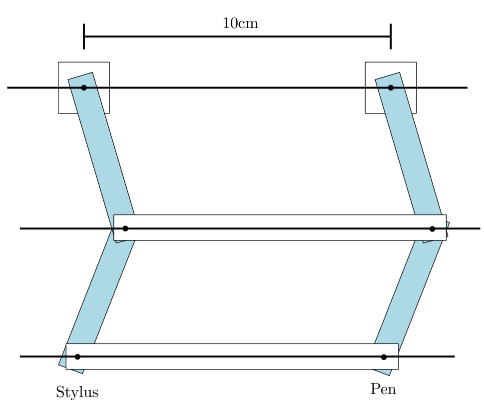
If we want to dilate by a factor of 3 about a, we need to make some changes to the machine, but the idea is the same — identify two similar triangles in the machine, but this time the bigger triangle will be enlarged by a factor of 3:



Because the two triangles differ by a factor of three this time, the distance between the center point and then pen will be *three* times the distance from the center point to the stylus.

Question 3:

To answer this question, we need to argue that, no matter how we move the stylus, the pen will always be 10cm directly to the right of the stylus. As in the previous problem, we need to identify some important parallel lines in this problem:



The three lines illustrated can move depending on the position of the stylus (some sketches may help to visualize this), but they will always remain parallel.

To see why, consider the edge lengths of the two quadrilaterals the lines form. Notice that in each quadrilateral, the opposite sides have equal lengths, so the quadrilaterals must be parallelograms. Notice that the two points that determine the topmost line are fixed to the tabletop, so the topmost line will always be horizontal. Hence, although the other two lines will move when we move the stylus, they will always remain horizontal as well (since they are parallel to the topmost line). Thus, the pen will always be 10cm horizontally to the right of the stylus.

As a historical aside, people (most notably Thomas Jefferson) used this type of machine in the 1700s and 1800s to copy their documents.