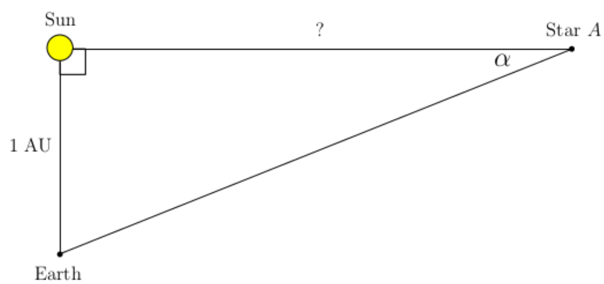
Question 1

The *parallax method* is a technique for estimating distances that has applications in astronomy. The diagram below depicts our earth, the sun, and a distant star A. Suppose you want to estimate the distance between our Sun and the star A marked in the picture.

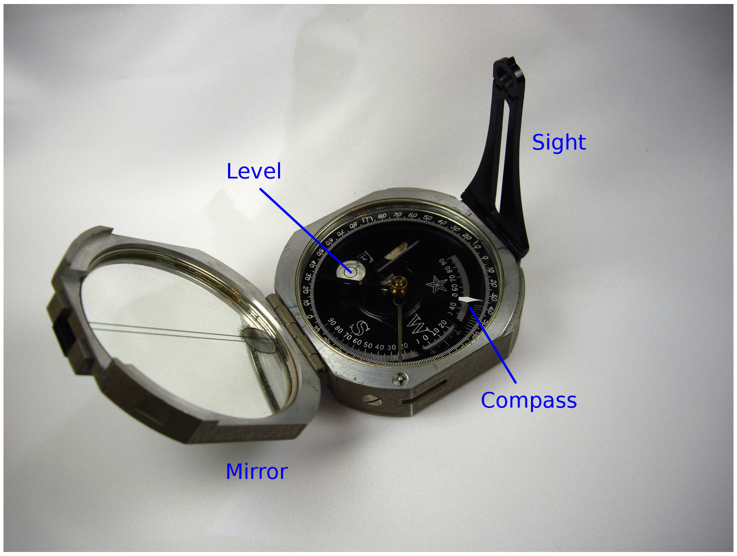


Part 1: Using your everyday knowledge of the solar system, discuss whether the diagram above is a scale model of the scenario we described. (You don't need to write down any formulas here, just think about how the distance from our earth to the sun compares with the distance from our sun to other stars.)

Part 2: Say you use a telescope to measure that angle α is 0.7611 arc-seconds (there are 3600 arc-second in one degree) and the distance between earth and the sun is 1.000 astronomical unit (AU). Use trigonometry to determine the distance from the sun to star A (in astronomical units). This is one of the main calculations astronomers perform when they use the parallax method!

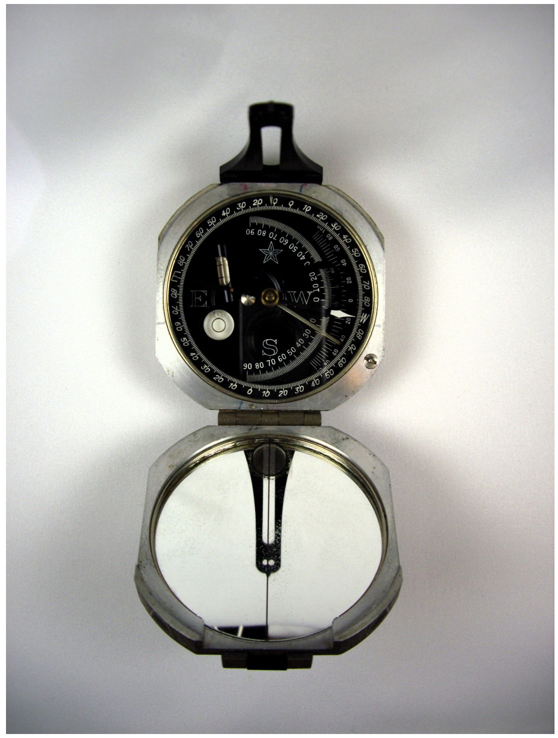
Question 2

A *pocket transit* is a special type of compass that can be used for measuring angles.



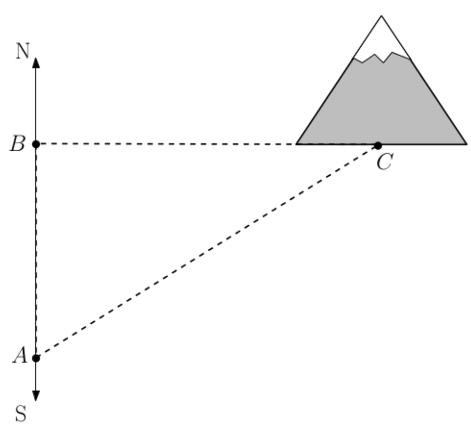
As you can see, this tool consists of a *compass*, a *mirror*, and a *sight*. The sight allows you to align the compass with an object in your field of vision. Here's one way to hold the pocket transit:

This is what you would see if you were holding the transit and looking down at the compass:



Do you see the reflection of the sight in the mirror? By looking into the mirror, you can align the sight with an object (like a tree) while keeping the compass horizontally level. Then, by looking at the compass needle, you can measure the angle between that object and magnetic north.

Suppose you're standing at point A on a North-South road. In the distance you see a mountain.



Using your pocket transit, you measure that at point A, the angle between point C and due north is 67 degrees. Then, you drive in your car 100 km north along the road to point B. At point B, your pocket transit tells you the angle between point C and due north is 90 degrees.

Part 1: Add the angles and lengths described in the story to the figure above.

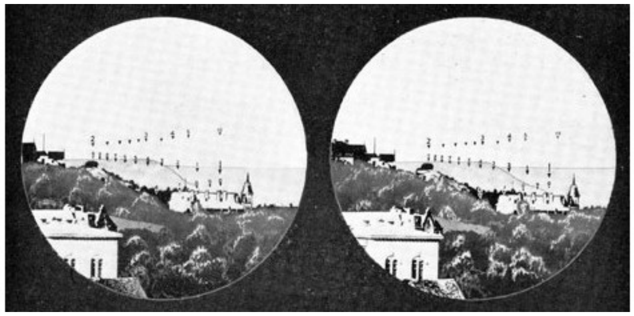
Part 2: How far is point B from point C? How far is point A from point C?

Question 3

In the first part of the twentieth century, sailors (and many other people) used an instrument called a *coincidence rangefinder* to determine the distance between their boat and another object, such as another vessel or a light house. Here's one example of a coincidence rangefinder:

Here is another larger version:

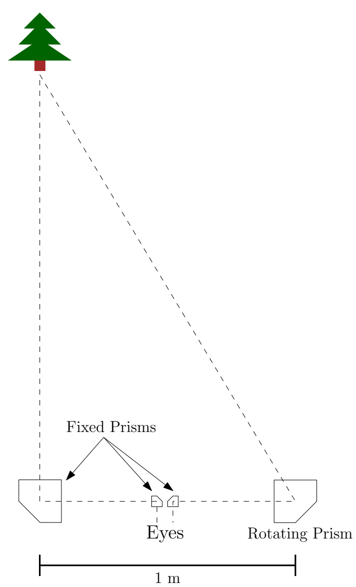
The machine worked like this: The sailor would look through the eyepieces and see two different images, one for each of the two lenses on the machine. Here's an example of what a sailor might see before adjusting the instrument:



As you can see, the two images are slightly different!

By adjusting a knob, the sailor could move one of two images horizontally (by rotating a prism) until the images aligned. By examining how far the knob needed to be rotated, the sailor could estimate the distance to the object in question.

Below is a schematic of the important parts of a coincidence rangefinder that's being used to measure the distance to a tree. You should assume that all of the angles in the figure are accurate.



Part 1: Do you see any important right triangles in this diagram? Indicate them by tracing their shape on the worksheet.

Part 2: Rotating the knob on the rangefinder rotates one of the prisms in the machine, which adjusts the view in one of the eyepieces. Suppose you've adjusted the knob so the views through the eyepieces match. What angle have you measured, in effect, by adjusting the knob? Indicate the angle on the diagram.

Part 3: Let's say the main prisms on the rangefinder are one meter apart and the important angle you measured in part 2 is called α. Give a formula that would allow you to calculate the distance between the rangefinder and the tree, in terms of α.

Worksheet Key

Disclaimer: Depending on what your students already know, these answers might be too brief or too thorough. I mostly intend for them to give you an idea how I approached these problems.

Answer 1: Part (i) is intended to point out the difference between a scale model and a diagram. The picture on the worksheet cannot be a scale model of the positions of the earth, sun, and a star — the distance from the earth and the sun is much, much smaller than the distance from the sun to another star. In other words, a scale model would be a **very** skinny triangle. That doesn't mean the diagram isn't useful, though! It still helps us to see the different lengths and angles in the problem, and how they relate to one another.

For part (ii), students will need to do a small conversion of units. Namely:

$$(0.7611 \text{ arc-seconds}) \cdot \frac{(1 \text{ degree})}{(3600 \text{ arc-seconds})} = 2.11417 \cdot 10^{-4} \text{ degrees}$$

depending on students' calculators, you might need to convert this into radians. If we call the distance we want to know L, we have the following formula:

$$\tan(\alpha) = \frac{1}{L}$$

Solving for L, we obtain:

$$L = \frac{1}{\tan(\alpha)} \approx 271,000 \text{ AU}$$

Answer 2:

Inexpensive pocket transits can be found online for about \$30.00 apiece. They can be used for a variety of educational "labs" outdoors, because they allow you to measure both vertical angles and horizontal angles. Once students know the law of cosines, one can assign a variety of interesting tasks, like measuring the heights of trees and mountains, the distance between mountains and buildings, etc.

To solve the problem in the text, students should fill in the two angles that appear on the road and label the distance along the road (100 km). With this setup, one can then use the tangent function to determine that the distance

d_{BC} from B to C satisfies the equation:

$$\tan(67°) = \frac{d_{BC}}{100}$$
$$d_{BC} = 100 \cdot \tan(67°) \approx 235\text{km}$$

To find the distance d_{AC} between A and C, students can either use the sine function or the Pythagorean theorem:

$$100^2 + d_{BC}^2 = d_{AC}^2$$
$$d_{AC} = \sqrt{100^2 + d_{BC}^2} \approx 255\text{km}$$

Answer 3:

In the first part, students should identify that there is a right angle attached to one of the larger prisms in the rangefinder. In the second part, they should identify that when the user rotates the movable prism in the rangefinder to align the two images, she is effectively measuring one of the (non-right) angles in the triangle. Since we also know the distance between the two lenses, we know the length of the base of the right triangle from part one. Hence, we can calculate the length L of the side of the triangle opposite angle α, since:

$$\frac{L}{1} = \tan(\alpha)$$